6.01

Lecture 12: Graph Search

6.01: Introduction to EECS I

The intellectual themes in 6.01 are recurring themes in engineering:
- design of complex systems
- modeling and controlling physical systems
- augmenting physical systems with computation
- building systems that are robust to uncertainty

Approach: focus on key concepts to pursue in depth

Module 1: Signals and Systems

Modeling and analyzing behavior of physical systems

Topics: Feedback Control Systems

Lab Exercises: Wall-finder, Wall-follower, Jousting
Module 2: Circuits
Designing, constructing, and analyzing physical systems

**Topics:** Resistive Networks, Op-Amps, Linearity and Equivalence

**Lab Exercises:** Design a new sensory modality for the robot

Module 3: Bayesian Reasoning
Modeling uncertainty and designing robust systems

**Topics:** Subjective Probability, Bayesian Inference

**Lab Exercises:** Localization and Parking

Module 4: Planning
Augmenting physical systems with computation.

**Topics:** Graph Search

**Lab Exercises:** Solving mazes, Path planning on maps
Graph Search (Path Planning)

What is a graph?
- Some set $V$ of vertices
- Some collection $E$ of edges connecting vertices

Example: 8-Puzzle

Start

1 2 3
4 5 6
7 8

Goal

1 2
3 4 5
6 7 8

Check Yourself

Start

1 2 3
4 5 6
7 8

Goal

1 2
3 4 5
6 7 8

How many different board configurations (states) exist?
1. $8^2 = 64$
2. $9^2 = 81$
3. $8! = 40,320$
4. $9! = 362,880$
5. None of the above
Graph Search

In this module:
- Develop algorithms to systematically "search" through a graph
- Analyze how well the algorithms perform
- Optimize the algorithms:
  - Find "better" paths (results)
  - Consider fewer cases (speed)
- Observe the algorithms at work in multiple contexts
  - Robot path planning
  - Route Planning in USA
  - Language
  - Biology

Example: Grid Search

Find path between 2 points on a rectangular grid.

Represent all possible paths from A with a tree:

Problem?

Notice that there are infinitely many paths.
The tree is infinitely large!

Strategy:
construct the tree incrementally while looking for a path
Python Representation of Grid

Represent the grid as instance of class `Grid`.

```python
class Grid:
    def __init__(self, width, height, start, goal):
        self.width = width
        self.height = height
        self.start = start
        self.goal = goal

grid = Grid(3, 3, (0,0), (2,2))
```

Search Trees in Python

Represent each node in the tree as an instance of `SearchNode`.

```python
class SearchNode:
    def __init__(self, state, parent):
        self.state = state
        self.parent = parent

    def path(self):
        p = []
        node = self
        while node:
            p.append(node.state)
            node = node.parent
        return p[::-1]
```

Note: no explicit representation for entire tree

Issues:
- need to “grow” the tree as we search it
- need to reconstruct paths in tree
Pathfinding Algorithm

Construct the tree and find a path to the goal.

Algorithm:
- Initialize agenda (list of nodes to consider)
- Repeat the following:
  - Remove one node from the agenda ("expand")
  - Add that node’s successors to the agenda ("visit")
until goal is found or agenda is empty
- Return resulting path

Order Matters!

Strategy: Replace last node in agenda by its successors

Agenda: A AB AD ADE ADG ADGH

Depth-first Search

Order Matters!

Strategy: Replace first node in agenda by its successors

Agenda: A AB ABA ABAB ABAD ABC ABE AD

Still Depth-first Search
Order Matters!

Strategy: Remove first node and add its successors to end

Agenda: A AB AD ABA ABC ABE ABD ADE ADG ABAB ABAD ABCB ABCF ABEB ABED ABEF ABEH ADAB ADAD ADED ADEF ADEH ADGD ADGH

Depth First Search (DFS):
- Push and Pop from same side of agenda
- Works down one branch of the tree before moving on to another branch

Breadth First Search (BFS):
- Push and Pop from different sides of agenda
- Considers all paths of length $n$ before considering paths of length $n+1$

Too Much Searching

Find path between 2 points on a rectangular grid.

Represent all possible paths with a tree:

But don’t need to consider all nodes!
Pruning

“Prune” the tree to reduce the amount of work.

**Pruning Strategy 1:**
Don’t consider any path the visits the same state twice.

**Algorithm:**
- Initialize **agenda** (list of nodes to consider)
- Repeat the following:
  - Remove one node from the agenda
  - Add each child (of that node) to the agenda **if its state is not in the parent’s path**
- until **goal is found** or **agenda is empty**
- Return resulting path

Pruning

Under strategy 1, BFS in the 3x3 grid still visits 16 nodes...
but there are only 9 states!

We should be able to reduce the search even further.
Dynamic Programming

Basic idea behind dynamic programming:
• Break big problem into easy ones, solve and combine.
• Remember the solutions to the easy problems for later use.

Appropriate if problem has:
• optimal substructure: best solution is combination of optimal solutions to sub-problems
• overlapping sub-problems: same sub-problem occurs many times while solving overall problem

As applies to search: (Depends slightly on which algorithm we’re using)

**BFS:** The shortest path \( S \rightarrow X \rightarrow G \) is made up of the shortest path \( S \rightarrow X \) and the shortest path \( X \rightarrow G \).

**DFS:** A path \( S \rightarrow X \rightarrow G \) is made up of a path \( S \rightarrow X \) and a path \( X \rightarrow G \).

The moral: once we have found a path \( S \rightarrow X \), we don’t need to spend time looking for other paths through \( X \).

Said another way: Many paths that include \( S \rightarrow X \), but don’t need to recompute while exploring rest of path (memoization); and once have a satisfactory path \( S \rightarrow X \), don’t need to keep looking for others (dynamic programming).

As applied to graph search: Don’t consider any path that visits a state that you have already visited via some other path.

Need to remember which states we have visited to avoid visiting them again.

Algorithm:
• Initialize **visited set**
• Initialize **agenda** (list of nodes to consider)
• Repeat the following:
  - Remove one node from the agenda
  - Add each child (of that node) to the agenda if its state is not already in the **visited set**, and add each of these new states to the **visited set**
  - until goal is found or agenda is empty
• Return resulting path
Check Yourself!

Consider a breadth-first search with dynamic programming, from A to I. How many states are visited?

1. 2
2. 4
3. 6
4. 8
5. 10

Search in lib601

lib601 procedure called search, takes arguments:
- successors: function that takes a state and returns a list of successor states
- start_state: the state from which to start the search
- goal_test: a function that takes a state and returns True if that state satisfies the goal condition, and False otherwise
- dfs: boolean; if True, run a depth-first search; if False, run a breadth-first search

search returns a list of states from the root of the tree to the goal, or None if no path exists.

Casting Problems as Search Problems

Biggest issue is choice of state.

From the state, we must be able to:
- Determine successors
- Test for goal condition

We'll get a lot of practice with this during the labs and homeworks this week.
Example: Grid Search

```python
class Grid:
    def __init__(self, width, height, start, goal):
        self.width = width
        self.height = height
        self.start = start
        self.goal = goal

grid = Grid(3, 3, (0,0), (2,2))
def grid_successors(state):
    r,c = state
    out = []
    for (dr,dc) in [(0,1),(1,0),(0,-1),(-1,0)]:
        if 0<=(r+dr)<grid.height and 0<=(c+dc)<grid.width:
            out.append((r+dr,c+dc))
    return out

result = search(grid_successors, grid.start, lambda x: x==grid.goal, False)
```

Recap

Developed two search algorithms:
- Breadth-first search
- Depth-first search

Discussed the benefits and drawbacks of each

Developed two pruning rules:
- Don’t consider paths that revisit states
- Only consider the first path to a given state

Labs This Week

Software Lab: Solving Mazes
Design Lab: Robots in Mazes